SCHOOL OF FUTURISTIC INTELLIGENCE

PRIVATE MATHEMATICS ASSIGNMENT

From Place-Value Switches to Recursive Fractions

Reconstructing finite and repeating expansions in variable bases

Student	Session	Suggested time
SFI Student	July 8, 2026	55 minutes

Complete this in two or three short sittings. Write the relevant place values above each numeral before calculating. Draw or describe subdivisions whenever that feels clearer than manipulating symbols. The goal is reconstruction and pattern recognition, not speed or rote memorization. Use the Ximals app only in the final section so that you first make your own predictions.

1. Rebuild the place-value picture

Annotate each numeral with powers or place values before evaluating it.

1. Label the seven positions in 1000.001_2, from left to right, with their place values.

2. Use the on/off method to convert 11010_2 to base 10.

3. Write 45 in binary by decomposing it into powers of two.

4. Evaluate 77_8, then state the next base-8 numeral and its base-10 value.

5. Continue the base-4 count 30_4, 31_4, __, __, __. Give the base-10 values of all five numerals.

2. Move through the point

Interpret each fractional digit as a reciprocal power of its base.

6. Convert 0.001_2 to a fraction.

7. Convert 0.101_2 to a fraction.

8. Convert 0.011_2 to a fraction.

9. Convert 0.01_8 to a fraction and explain the two subdivisions involved.

10. Write 1/2 as a one-digit terminating expansion in bases 2, 4, 6, 8, 10, and 12.

11. Write 1/4 in bases 2, 4, 8, and 12 using a terminating expansion.

3. See why repetition is exact

Use either a recursive area explanation or the indicated place-value shift. Do not describe the result as an approximation.

12. Describe how you would draw the first four levels of $0.1111\ldots_{10}$. List the four shaded quantities.

13. Complete and explain the pattern: $0.111\ldots_4 = \_\_$, $0.111\ldots_6 = \_\_$, and $0.111\ldots_8 = \_\_$.

14. Let $x = 0.010101\ldots_2$. Shifting two binary places gives $4x = 1.010101\ldots_2$. Use the repeated tail to solve for x.

15. The video showed $1/5 = 0.00110011\ldots_2$. Let x equal this expansion and shift four binary places. What equation results, and what value does it give?

16. Explain $0.111\ldots_3 = 1/2$ in two ways: through its area pattern and through a one-place shift.

4. Predict, observe, and contemplate

Use the Ximals app if available. Press the spacebar to enter manual mode. If the app is unavailable, sketch recursive bars or rectangles instead.

17. Before checking, predict the representations of 1/3 in bases 2, 4, and 12. Then compare the three visual patterns.

18. Choose either 1/3 in binary or 1/2 in base 3. Record three observations: how each level is subdivided, what repeats, and why the total shaded area remains the stated fraction.

19. Look at your chosen pattern quietly for two minutes, then redraw it from memory and write one sentence identifying what stays structurally unchanged as the diagram zooms inward.